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GENERALIZED ALTERING DISTANCES AND FIXED POINT FOR OCCASIONALLY HYBRID MAPPINGS

Abstract. In this work we are interested in the generalization of the result which is in the article [20]. To realize this, we weaken the two conditions that are : weakly altering distance and occasionally weakly compatible, then we neglect the distance altered because of some changes in theorem.

Key words: symmetric space,($P_{(n,m)}$), (P^*), occasionally weakly compatible, hybrid mappings, altering distance, common fixed point.

AMS Mathematics Subject Classification: 54H25, 47H10

1. Introduction

Let X be a nonempty set. A symmetric on X is a nonnegative real valued function d on $X \times X$ such that:

(i) $d(x, y) = 0$ iff $x = y$,

(ii) $d(x, y) = d(y, x)$ $\forall x, y \in X$.

Let (X, d) be a metric (symmetric) space and $B(X)$ the set of all nonempty bounded subset of X. As in [5], [6] we define the functions $\delta(A, B)$ and $D(A, B)$, where $A, B \in B(X)$:

$$D(A, B) = \inf\{d(a, b)|a \in A, b \in B\},$$

$$\delta(A, B) = \sup\{d(a, b)|a \in A, b \in B\}.$$

If $A = \{a\}$ then $\delta(A, B) = \delta(a, B)$. If $A = \{a\}$ and $B = \{b\}$ then $\delta(A, B) = d(a, b)$. If follows immediately from the definition of δ that:

$$\delta(A, B) = \delta(B, A), \quad \forall A, B \in B(X).$$

If $\delta(A, B) = 0$ then $A = B = \{a\}$.

Definition 1. *The hybrid pair* $f : X \longrightarrow X$ *and* $F : X \longrightarrow B(X)$ *is occasionally weakly compatible (owc)* [1] *if there exists* $x \in X$ *such that* $fx \in Fx$ *and* $fFx \subset Ffx$.

Definition 2. *Let* $\mathcal{F}_{\mathcal{W}}$ *be the set of all functions* $\phi : \mathbb{R}_+^6 \longrightarrow \mathbb{R}$ *satisfying the following conditions :*
(ϕ_1) ϕ *is nonincreasing in variables* t_2, t_5 *and* t_6,
(ϕ_2) $\phi(t,t,0,0,t,t) \geq 0$, $\forall t > 0$.

Example 1. $\phi(t_1, \dots, t_6) = t_1 - \max\{t_2, \frac{1}{2}(t_3 + t_4), \frac{1}{2}(t_5 + t_6)\}$.

Example 2. $\phi(t_1, \dots, t_6) = t_1 - h\max\{t_2, t_3, t_4, \frac{1}{2}(t_5 + t_6)\}$, where $h \in]0, 1[$.

Definition 3. *A weakly altering distance is a mapping* $\psi : [0, +\infty[\longrightarrow [0, +\infty[$ *which satisfies:*
(i) ψ *is increasing,*
(ii) $\psi(t) = 0$ *if and only if* $t = 0$.

Theorem 1 ([20]). *Let* f, g *be self maps of the symmetric space* (X, d) *and* F, G *be maps of* X *into* $B(X)$ *such that the pair* (f, F), (g, G) *are owc. If*

$$\phi(\psi(\delta(Fx, Gy)), \psi(d(f(x), g(y))), \psi(D(Fx, fx)), \psi(D(g(y), Gy)), \psi(\delta(f(x), Gy)), \psi(\delta(g(y), Fx))) < 0 \tag{1}$$

for all $x, y \in X$ *for which* $f(x) \neq g(y)$ *where* $\psi(t)$ *is a weakly altering distance and* $\phi \in \mathcal{F}_{\mathcal{W}}$, *then* f, g, F *and* G *have a unique common fixed point.*

2. Generalized weakly altering distance and owc

Definition 4. *The pair* $f : X \longrightarrow X$ *and* $F : X \longrightarrow 2^X$, *satisfies* $(P_{n,m})$ *if* $\exists\ x \in X$ *such that* $f^m x \in Fx$ *and* $f^n x \in (Ff^{n-m}x) \cap (Ff^m x)$, *with* $n, m \in \mathbb{N}$ *and* $n > m$. $(f^0 x = x)$.

Remark 1. If f and F are owc, then (f, F) satisfies $(P_{2,1})$.

Example 3. Let $f : [0,1] \longrightarrow [0,1]$ and $F : [0,1] \longrightarrow B([0,1])$, such that

$$f(x) = \begin{cases} 1 \text{ if } x \in \{0,1\} \\ 0 \text{ else} \end{cases} \quad \text{and} \quad Fx = \begin{cases}]0,1] \text{ if } x \in \{0,1\} \\ 0 \text{ else} \end{cases}$$

then $f(0) \in F0$ and $f^3(0) \in (Ff^2(0)) \cap (Ff(0))$, so (f, F) satisfies $(P_{3,1})$.

Definition 5. *Let $\psi_i : [0,+\infty[\longrightarrow [0,+\infty[$, $i = 1,\ldots,6$ we say that ψ_i satisfies (P^*) if : $\forall\, t > 0$, $\forall\, j = 2, 5, 6$, $\psi_1(t) \geq \psi_j(t)$, ψ_j is increasing, $\psi_1(t) > 0$, and $\psi_3(0) = \psi_4(0) = 0$.*

Remark 2. A weakly altering distance satisfies (P^*).

Example 4. Let $\psi_i : [0,+\infty[\longrightarrow [0,+\infty[$, $i = 1,...,6$, such that :

$$\psi_1(t) = te^t, \quad \psi_2(t) = t^3, \quad \psi_3(t) = \sin^2 t,$$
$$\psi_4(t) = t^2, \quad \psi_5(t) = t, \quad \psi_6(t) = \frac{t^2}{24}.$$

3. Main results

Our motivation for the next result is to show that f, g, F and G may not have a common fixed point, but their iterates (or some of them) can have it. (see the example below).

Theorem 2. *Let $f, g : X \longrightarrow X$ and $F, G : X \longrightarrow B(X)$ such that the pair (f, F), satisfies (P_{n_1,m_1}), and (g, G) satisfies (P_{n_2,m_2}). If*

$$\begin{cases} \forall (x,y) \in \{(a,b) \in X \times X, | f^{m_1}a \neq g^{m_2}b, \}, \exists \phi \in \mathcal{F}_{\mathcal{W}}, \text{ such that} \\ \phi(\delta(Fx,Gy), d(f^{m_1}x, g^{m_2}y), D(f^{m_1}x, Fx), D(g^{m_2}y, Gy), \\ \qquad\qquad \delta(f^{m_1}x, Gy), \delta(Fx, g^{m_2}y)) < 0 \end{cases}$$

then $f^{n_1-m_1}, g^{n_2-m_2}, F$ and G have a unique common fixed point.

Proof. Since (f, F) satisfies (P_{n_1,m_1}), and (g, G) satisfies (P_{n_2,m_2}), there exists $x, y \in X$ such that $f^{m_1}x \in Fx$, $g^{m_2}y \in Gy$, $f^{n_1}x \in (Ff^{n_1-m_1}x) \cap (Ff^{m_1}x)$ and $g^{n_2}y \in (Gg^{n_2-m_2}y) \cap (Gg^{m_2}y)$. We prove that $f^{m_1}x = g^{m_2}y$. Suppose that $f^{m_1}x \neq g^{m_2}y$, then $0 < d(f^{m_1}x, g^{m_2}y) \leq \delta(Fx, Gy)$, so we deduce by (2) and (ϕ_1) that

$$\phi(\delta(Fx,Gy), \delta(Fx,Gy), 0, 0, \delta(Fx,Gy), \delta(Fx,Gy)) < 0,$$

witch is a contradiction of (ϕ_2). Next we show that $f^{m_1}x = f^{n_1}x$. Suppose that $f^{m_1}x \neq f^{n_1}x$, then $0 < d(f^{n_1}x, f^{m_1}x) \leq \delta(Ff^{n_1-m_1}x, f^{m_1}x) = \delta(Ff^{n_1-m_1}x, g^{m_2}y) \leq \delta(Ff^{n_1-m_1}x, Gy)$, so by (2) and (ϕ_1) we obtain $\phi(\delta(Ff^{n_1-m_1}x, Gy), d(f^{n_1}x, g^{m_2}y), 0, 0, \delta(f^{n_1}x, Gy), \delta(Ff^{n_1-m_1}x, g^{m_2}y)) < 0$ and $\phi(\delta(Ff^{n_1-m_1}x, Gy), \delta(Ff^{n_1-m_1}x, Gy), 0, 0, \delta(Ff^{n_1-m_1}\, x, Gy), \delta(Ff^{n_1-m_1}\, x, Gy)) < 0$, which is a contradiction of (ϕ_2). Hence, $f^{m_1}x = f^{n_1}x$. we have also, $g^{m_2}y = g^{n_1}y$. Consequently we deduce that $f^{n_1-m_1}f^{m_1}x = f^{m_1}x = g^{m_2}y = g^{n_2}y = g^{n_2-m_2}f^{m_1}x$, so $f^{m_1}x$ is a common fixed point of $f^{n_1-m_1}$ and $g^{n_2-m_2}$. On the other hand $f^{m_1}x = f^{n_1}x \in Ff^{m_1}x$, and $f^{m_1}x$ is a fixed point of F. Similarly, $f^{m_1}x = g^{m_2}y = g^{n_2}y \in Gg^{m_2}y = Gf^{m_1}x$, and $f^{m_1}x$ is a fixed point of G.

Consequently $w = f^{m_1}x$ is a common fixed point of $f^{n_1-m_2}, g^{n_2-m_2}, F$ and G. Now we show that w is unique. Suppose that $w' \neq w$ is an other common fixed point of $f^{n_1-m_2}, g^{n_2-m_2}, F$ and G. Because $0 < d(w,w') = d(f^{m_1}w, g^{m_2}w') \leq \delta(Fw, Gw')$, there exists $\phi \in \mathcal{F}_{\mathcal{W}}$, such that $\phi(\delta(Fw,Gw'), d(f^{m_1}w, g^{m_2}w'), D(f^{m_1}w, Fw), D(g^{m_2}w', Gw'), \delta(f^{m_1}w, Gw'), \delta(Fw, g^{m_2}w')) < 0$. By (2) and (ϕ_1) deduce that $\phi(\delta(Fw,Gw'), \delta(Fw,Gw'), 0, 0, \delta(Fw,Gw'), \delta(Fw,Gw')) < 0$, which is a contradiction of (ϕ_2). so $w = f^{m_1}x$ is the unique common fixed point of $f^{n_1-m_2}, g^{n_2-m_2}, F$ and G. ■

Corollary 1. *For $n = 2$, $m = 1$, $\psi(.)$ is a weakly altering distance and $\phi \in \mathcal{F}_{\mathcal{W}}$, let $\overline{\phi}(t_1,t_2,t_3,t_4,t_5,t_6) = \phi(\psi(t_1),\psi(t_2),\psi(t_3),\psi(t_4),\psi(t_5),\psi(t_6))$, so it is clear that $\overline{\phi} \in \mathcal{F}_{\mathcal{W}}$, then by theorem 2 we obtain Theorem 1.*

Corollary 2. *Let $f, g : X \longrightarrow X$ and $F, G : X \longrightarrow B(X)$ such that the pair (f, F), satisfies (P_{n_1,m_1}), and (g, G) satisfies (P_{n_2,m_2}). If*

$$(2)\qquad \begin{cases} \forall (x,y) \in \{(a,b) \in X \times X, | f^{m_1}a \neq g^{m_2}b, \}, \exists (\psi_i)_{1 \leq i \leq 6} \\ \textit{which satisfies } (P^*) \textit{ and } \phi \in \mathcal{F}, \quad \textit{such that} \\ \phi(\psi_1(\delta(Fx,Gy)), \psi_2(d(f^{m_1}x, g^{m_2}y)), \psi_3(D(f^{m_1}x, Fx)), \\ \psi_4(D(g^{m_2}y, Gy)), \psi_5(\delta(f^{m_1}x, Gy)), \psi_6(\delta(Fx, g^{m_2}y))) < 0 \end{cases}$$

then $f^{n_1-m_1}, g^{n_2-m_2}, F$ and G have a unique common fixed point.

Proof. Let $\overline{\phi}(t_1,t_2,t_3,t_4,t_5,t_6) = \phi(\psi_1(t_1),\psi_2(t_2),\psi_3(t_3),\psi_4(t_4),\psi_5(t_5),\psi_6(t_6))$, then it is clear that $\overline{\phi} \in \mathcal{F}_{\mathcal{W}}$. So (3) $\Longrightarrow$ (2). ■

Example 5. Let $X = [0,12]$, $d(x,y) = (x-y)^2$, and

$$Fx = \begin{cases} \{1\} & \text{if } x \in]0,2[, \\ \{0\} \cup \{\frac{1}{4}\} & \text{if } x \in \{0\} \cup [2,12] \end{cases} \qquad f(x) = \begin{cases} 2 & \text{if } x = 0, \\ 0 & \text{if } x = 1, \\ 1 & \text{if } x = 2, \\ 10 & \text{if } x = 12 \\ x+8 & \text{if } x \in]0,1[\cup]1,2[, \\ 12 & \text{if } x \in]2,12[, \end{cases}$$

$$Gx = \begin{cases} \{0\} & \text{if } x \in [0,2], \\ [1,4] & \text{if } x \in]2,12] \end{cases} \qquad g(x) = \begin{cases} 0 & \text{if } x = 0, \\ 10 & \text{if } x = 12 \\ x+3 & \text{if } x \in]0,2], \\ 12 & \text{if } x \in]2,12[, \end{cases}.$$

We have $f(0) \in F0$, $f^4(0) \in Ff^3(0) \cap Ff(0)$, $g(0) \in G0$ and $g^2(0) \in Gg(0)$, so (f, F) satisfies $(P_{(4,1)})$ and (g, G) satisfies $(P_{(2,1)})$. Put

$$R = \delta(Fx, Gy) - \max\{d(f(x), g(y)), \frac{1}{2}[D(f(x), Fx) + D(g(y), Gy)], \frac{1}{2}[\delta(f(x), Gy) + \delta(g(y), Fx)]\},$$

then we have the following situations:

1) If $x = 0$ and $y \in [0, 2]$, we get $f(x) \neq g(y)$ and

$$\begin{aligned} \delta(Fx, Gy) &= \frac{1}{16} \\ &< d(f(x), g(y)) \\ &\leq \max\{d(f(x), g(y)), \frac{1}{2}[D(f(x), Fx) + D(g(y), Gy)], \\ &\qquad \frac{1}{2}[\delta(f(x), Gy) + \delta(g(y), Fx)]\}. \end{aligned}$$

2) If $x = 0$ and $y \in]2, 12]$, we get $f(x) \neq g(y)$ and

$$\begin{aligned} \delta(Fx, Gy) &= 16 \\ &< \frac{1}{2}[\delta(f(x), Gy) + \delta(g(y), Fx)] \\ &\leq \max\{d(f(x), g(y)), \frac{1}{2}[D(f(x), Fx) + D(g(y), Gy)], \\ &\qquad \frac{1}{2}[\delta(f(x), Gy) + \delta(g(y), Fx)]\}. \end{aligned}$$

3) If $x \in]0, 1[\cup]1, 2[$ and $y = 0$, we get $f(x) \neq g(y)$ and

$$\begin{aligned} \delta(Fx, Gy) &= 1 \\ &< \frac{(x+8)^2 + 1}{2} \\ &= \frac{1}{2}[\delta(f(x), Gy) + \delta(g(y), Fx)] \\ &\leq \max\{d(f(x), g(y)), \frac{1}{2}[D(f(x), Fx) + D(g(y), Gy)], \\ &\qquad \frac{1}{2}[\delta(f(x), Gy) + \delta(g(y), Fx)]\}. \end{aligned}$$

4) If $x \in]0,1[\cup]1,2[$ and $y \in]0,2]$, we get $f(x) \neq g(y)$ and

$$\begin{aligned}\delta(Fx,Gy) &= 1\\ &< (x-y+5)^2\\ &= d(f(x),Gy)\\ &\le \max\{d(f(x),g(y)),\frac{1}{2}[D(f(x),Fx)+D(g(y),Gy)],\\ &\qquad \frac{1}{2}[\delta(f(x),Gy)+\delta(g(y),Fx)]\}.\end{aligned}$$

5) If $x \in]0,1[\cup]1,2[$ and $y \in]2,12]$, we get $f(x) \neq g(y)$ and

$$\begin{aligned}\delta(Fx,Gy) &= 9\\ &< \frac{D(g(y),Gy)}{2}\\ &\le \frac{1}{2}[D(f(x),Fx)+D(g(y),Gy)]\\ &\le \max\{d(f(x),g(y)),\frac{1}{2}[D(f(x),Fx)+D(g(y),Gy)],\\ &\qquad \frac{1}{2}[\delta(f(x),Gy)+\delta(g(y),Fx)]\}.\end{aligned}$$

6) If $x = 1$ and $y \in]0,2]$, we get $f(x) \neq g(y)$ and

$$\begin{aligned}\delta(Fx,Gy) &= 1\\ &< d(f(x),g(y))\\ &\le \max\{d(f(x),g(y)),\frac{1}{2}[D(f(x),Fx)+D(g(y),Gy)],\\ &\qquad \frac{1}{2}[\delta(f(x),Gy)+\delta(g(y),Fx)]\}.\end{aligned}$$

7) If $x = 1$ and $y \in]2,12]$, we get $f(x) \neq g(y)$ and

$$\begin{aligned}\delta(Fx,Gy) &= 9\\ &< d(f(x),g(y))\\ &\le \max\{d(f(x),g(y)),\frac{1}{2}[D(f(x),Fx)+D(g(y),Gy)],\\ &\qquad \frac{1}{2}[\delta(f(x),Gy)+\delta(g(y),Fx)]\}.\end{aligned}$$

8) If $x = 2$ and $y \in [0, 2]$, we get $f(x) \neq g(y)$ and

$$\begin{aligned}\delta(Fx, Gy) &= \frac{1}{16} \\ &< d(f(x), g(y)) \\ &\leq \max\{d(f(x), g(y)), \frac{1}{2}[D(f(x), Fx) + D(g(y), Gy)], \\ &\qquad \frac{1}{2}[\delta(f(x), Gy) + \delta(g(y), Fx)]\}.\end{aligned}$$

9) If $x = 2$ and $y \in]2, 12]$, we get $f(x) \neq g(y)$ and

$$\begin{aligned}\delta(Fx, Gy) &= \frac{1}{16} \\ &< d(f(x), g(y)) \\ &\leq \max\{d(f(x), g(y)), \frac{1}{2}[D(f(x), Fx) + D(g(y), Gy)], \\ &\qquad \frac{1}{2}[\delta(f(x), Gy) + \delta(g(y), Fx)]\}.\end{aligned}$$

10) If $x \in]2, 12]$ and $y \in [0, 2]$, we get $f(x) \neq g(y)$ and

$$\begin{aligned}\delta(Fx, Gy) &= \frac{1}{16} \\ &< d(f(x), g(y)) \\ &\leq \max\{d(f(x), g(y)), \frac{1}{2}[D(f(x), Fx) + D(g(y), Gy)], \\ &\qquad \frac{1}{2}[\delta(f(x), Gy) + \delta(g(y), Fx)]\}.\end{aligned}$$

All the conditions of theorem 2 are satisfied with ϕ as in example 1, then 0 is the unique common fixed point of f^3, g, F and G, but it is not a common fixed point of f, g, F and G.

4. Applications

Definition 6. *We say that $h \in \mathcal{E}_{\mathcal{W}}$, if $h : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is locally integrable on $[0, +\infty[$ and satisfies $\int_0^\epsilon h(t)dt > 0$ for $\epsilon > 0$.*

Lemma 1. *The function $\psi(x) = \int_0^x h(t)dt$, were $h \in \mathcal{E}_{\mathcal{W}}$ is an altering distance.*

Theorem 3. *Let $f, g : X \longrightarrow X$ and $F, G : X \longrightarrow B(X)$ such that the pair (f, F), satisfies (P_{n_1,m_1}), and (g, G) satisfies (P_{n_2,m_2}). If*

$$(3)\begin{cases} \forall (x,y) \in \{(a,b) \in X \times X, | f^{m_1}a \neq g^{m_2}b, \}, \exists \phi \in \mathcal{F}_{\mathcal{W}} \text{ and} \\ (h_i)_{1\leq i\leq 6} \subset \mathcal{E}_{\mathcal{W}} \text{ with } h_1 \geq h_i (i = 2,5,6) \text{ such that} \\ \phi\Big(\int_0^{\delta(Fx,Gy)} h_1(t)dt, \int_0^{d(f^{m_1}x,g^{m_2}y)} h_2(t)dt, \int_0^{D(f^{m_1}x,Fx)} h_3(t)dt, \\ \int_0^{D(g^{m_2}y,Gy)} h_4(t)dt, \int_0^{\delta(f^{m_1}x,Gy)} h_5(t)dt, \int_0^{\delta(Fx,g^{m_2}y)} h_6(t)dt\Big) < 0 \end{cases}$$

then $f^{n_1-m_1}$, $g^{n_2-m_2}$, F and G have a unique common fixed point.

Proof. As in Lemma 1 we have

$$\begin{aligned} \psi_1(\delta(Fx,Gy)) &= \int_0^{\delta(Fx,Gy)} h_1(t)dt \\ \psi_2(d(f^{m_1}x,g^{m_2}y)) &= \int_0^{d(f^{m_1}x,g^{m_2}y)} h_2(t)dt \\ \psi_3(D(f^{m_1}x,Fx)) &= \int_0^{D(f^{m_1}x,Fx)} h_3(t)dt \\ \psi_4(D(g^{m_2}y,Gy)) &= \int_0^{D(g^{m_2}y,Gy)} h_4(t)dt \\ \psi_5(\delta(f^{m_1}x,Gy)) &= \int_0^{\delta(f^{m_1}x,Gy)} h_5(t)dt \\ \psi_6(\delta(Fx,g^{m_2}y)) &= \int_0^{\delta(Fx,g^{m_2}y)} h_6(t)dt. \end{aligned}$$

Then, by (5), we have

$$\begin{cases} \forall (x,y) \in \{(a,b) \in X \times X, | f^{m_1}a \neq g^{m_2}b, \}, \exists (\psi_i)_{1\leq i\leq 6} \text{ which satisfies } (P^*) \\ \text{and } \phi \in \mathcal{F}, \text{ such that} \\ \phi(\psi_1(\delta(Fx,Gy)), \psi_2(d(f^{m_1}x,g^{m_2}y)), \psi_3(D(f^{m_1}x,Fx)), \\ \qquad \psi_4(D(g^{m_2}y,Gy)), \psi_5(\delta(f^{m_1}x,Gy)), \psi_6(\delta(Fx,g^{m_2}y))) < 0 \end{cases}$$

The conditions of Corollary 2 are satisfied, so theorem 3 follows from Corollary 2. ■

For example, by Theorem 3 we obtain.

Corollary 3. *Let $f, g : X \longrightarrow X$ and $F, G : X \longrightarrow B(X)$ such that the pair (f, F), satisfies (P_{n_1,m_1}), and (g, G) satisfies (P_{n_2,m_2}). If*

$$\begin{cases} \forall (x,y) \in \{(a,b) \in X \times X, | f^{m_1}a \neq g^{m_2}b, \}, \exists \phi \in \mathcal{F}_{\mathcal{W}} \text{ and } h \in \mathcal{E}_{\mathcal{W}} \\ \text{such that} \\ \int_0^{\delta(Fx,Gy)} h(t)dt < \max\{\int_0^{d(f^{m_1}x,g^{m_2}y)} h(t)dt, \frac{1}{2}[\int_0^{D(f^{m_1}x,Fx)} h(t)dt \\ \qquad + \int_0^{D(g^{m_2}y,Gy)} h(t)dt], \frac{1}{2}[\int_0^{\delta(f^{m_1}x,Gy)} h(t)dt + \int_0^{\delta(Fx,g^{m_2}y)} h(t)dt]\} \end{cases}$$

then $f^{n_1-m_1}$, $g^{n_2-m_2}$, F and G have a unique common fixed point.

Proof. It is a consequence of theorem 3 by taking $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \max\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\}$ and $h_1 = h_2 = \ldots h_6 = h \in \mathcal{E}_{\mathcal{W}}$. ■

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